ONE- AND TWO-DEGREE-OF-FREEDOM VIBRO-IMPACT DYNAMICS CONTROL

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Abstract. *In this work a control of one- and two-degrees-of-freedom vibro-impact system with a delay loop is presented and analysed. The aim of the delay loop application is to make the return to the vibro-impact periodic motion after the occurrence of disturbances quicker than in the case without the loop. The proposed analytical approach yields the required delay loop coefficients. The original vibro-impact map is described which allows to solve the problem analytically for near resonance case in one-degree-of-freedom system. The numerical calculations have proved our theoretical investigations. In addition, an efficient delay loop control applied to two-degree-of-freedom system is proposed.*

1 INTRODUCTION

Vibro-impact vibration problems with one-degree-of-freedom systems have a long history in mechanics. The problems like stationary subharmonic motions and their stability, the influence of damping and friction on vibro-impact dynamics, elastic and plastic type impacts, time histories and phase portraits of the vibro-impact systems have been considered [1-7].

We have to mention also the chaotic dynamics of the vibro-impact one-degree-of-freedom oscillators. A vertical motion of a ball acting on sinusoidally vibrating table has been analysed. It has been investigated using a simple model of difference equations satisfying an impact condition. For small velocities and for a restitution coefficient close to value one, a chaotic dynamics has been found. The mentioned simple systems have been also investigated experimentally. It has been observed, that for a restitution coefficient equal zero a velocity of the reflected body was also equal zero which allows one to construct a one dimensional map. In addition, two dimensional maps, as the singular perturbations of the one dimensional, have been analysed.

The existence and stability of the vibro-impact periodic motion have been investigated [12]. The intervals of parameters, for which the stable vibro-impact motion does not appear, have been numerically detected [8, 9, 13]. For some of those parameters, the bifurcation processes leading to chaotic dynamics have been observed. In reference [14] the vibro-imact chaotic dynamics using the Duffing [15] oscillator has been reported.

Recently more attention is paid to the analysis of the chaotic vibro-impact dynamics in one- and two-degree-of-freedom oscillators with various friction models.

The peculiar bifurcation governing a transition between vibration with and without impacts has been analysed (the so called "grazing bifurcation") [16]. The effect of the folded vibro-impact rotating bodies on the periodic and chaotic dynamics has been reported [17]. An interesting proposition of the vibro-impact dynamics using the catastrophe theory has been outlined [18].

The vibro-impact systems are very important in industry. It turned out that purely theoretical impact models governed by an artificial rule between the velocities just before and just after an impact (joint via the restitution coefficient) had not been satisfactory confirmed with the experiments.

Nowadays, in the field of the vibro-impact phenomena two main directions of investigations are dominating. The first one concerns the mathematical description of a vibro-impact motion including the restitution coefficient. The experimental investigation shows that it depends on many parameters such as material of impacting bodies, their shapes and velocities [19, 20] and therefore it is difficult to define it exactly.

The second direction concerns the control of vibro-impact systems. Many references are devoted to a field of control of the nonlinear systems including the control of chaotic orbits [21, 22]. Among them we mention only the control methods with a delay loop [23], the adaptive control [24], the learning control systems [26] and others[25].

Generally, the aim of those approaches is to control rather complicated systems where mathematical model is not known and their dynamics is tracking numerically. In contrary to those methods, in this work we propose an analytical approach to determine suitable delay loop coefficients to realise the required vibro-impact periodic dynamics for both resonance and non-resonance cases. The obtained analytical formulas allow for a proper choice of the delay loop coefficients in order to achieve the required vibro-impact periodic motion quicker then in the case without a loop. When the vibro-impact periodic motion is achieved the delay loop is automatically switched off.

2 THE ANALYSED SYSTEM

The analysed system with the kinematic excitation is presented in figure 1. The system dynamics (including a delay loop) is governed by the equation:

$$\ddot{x} + c\dot{x} + \alpha^2 x = P_0 \cos\omega t + S[x(t) - x(t-T)] + Q[\dot{x}(t) - \dot{x}(t-T)] \qquad \text{for} \quad x < s, \tag{1}$$
$$x_+ = x_-, \quad \dot{x}_+ = -R\dot{x}_- \qquad \text{for} \quad x \geq s,$$

where: $c = c_1/m$, $\alpha^2 = (k_1 + k_2)/m$, $P_0 = k_2 y_0/m$, $S = k_2 p/m$, $Q = k_2 q/m$ and $T = 2\pi k/\omega$ is the period of a stabilised periodic orbit, R denotes the restitution coefficient, and s is the constraint. The natural number k defines the number of excitation periods occurring during one impact. The indeces "+" and "–" define positions and velocities of the body just after and before an impact, correspondingly.

(a) (b)

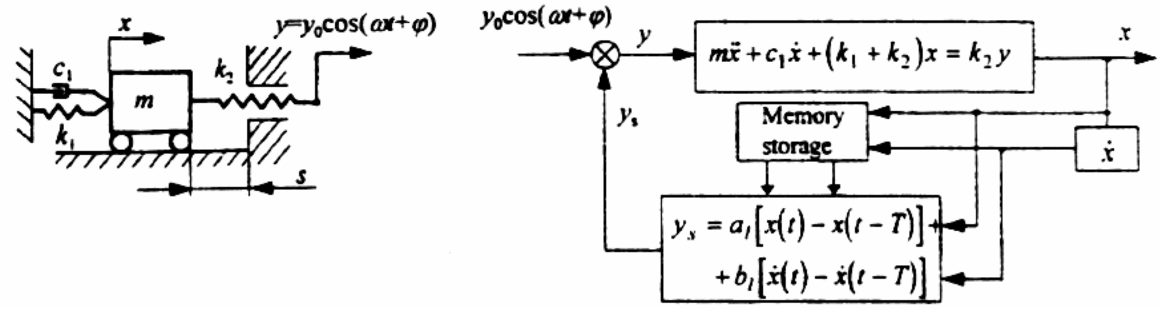


Figure 1. One-degree-of-freedom vibro-impact system kinematically excited (a) and the control diagram (b)

It is assumed (see figure 1) that the stabilised periodic orbit $x_0(t) = x_0(t-T)$ has the period of the excitation the same as in the system without that loop and that $x_0(t)$ is the particular solution for the system with and without the delay loop [27-29]. The delay loop starts to operate when the disturbances occur and is going to act on the dynamics in order to achieve the vibro-impact periodic motion quicker than in the case without a loop.

2.1 Periodic motion

Consider the periodic vibrations of the analysed system without the delay loop. Then, the vibro-impact dynamics (1) can be reduced to the following form:

$$\begin{aligned} &\ddot{x} + c\dot{x} + \alpha^2 x = P_0 \cos\omega t \qquad \text{for} \quad x < s, \\ &x_+ = x_-, \quad \dot{x}_+ = -R\dot{x}_- \qquad \text{for} \quad x \geq s. \end{aligned} \tag{2}$$

The solution to the equation (2) has the form:

$$\begin{aligned} &x(t) = \mathrm{e}^{-0.5ct}\left(C\cos\lambda_1 t + D\sin\lambda_1 t\right) + F_1\cos(\omega t + \theta) \quad \text{for} \quad x < s, \\ &x_+ = x_-, \quad \dot{x}_+ = -R\dot{x}_- \qquad \text{for} \quad x \geq s, \end{aligned} \tag{3}$$

where:

$$F_1 = \frac{P_0}{\sqrt{\left(\alpha^2 - \omega^2\right) + c^2\omega^2}}, \qquad \lambda_1 = \sqrt{\alpha^2 - \left(\frac{c}{2}\right)^2},$$

and the coefficients C, D, and θ are defined using the impact boundary condition. The boundary conditions have the form:

$$\begin{aligned} &t = 0, \quad x(t) = s, \quad \dot{x}(t) = \dot{x}_+, \\ &t = \frac{2\pi k}{\omega} = 2\beta, \quad x(t) = s, \quad \dot{x}(t) = \dot{x}_-. \end{aligned} \tag{4}$$

Differentiating the equation (3) and taking into acount the boundary conditions (4) we get the following four equations:

$$C + F_1\cos\theta = s,$$

$$-0.5cC + \lambda_1 D - F_1\omega\sin\theta = \dot{x}_+,$$

$$e^{-\beta c}\left(C\cos 2\beta\lambda_1 + D\sin 2\beta\lambda_1\right) + F_1\cos\theta = s,$$

$$-0.5ce^{-\beta c}\left(C\cos 2\beta\lambda_1 + D\sin 2\beta\lambda_1\right) + e^{-\beta c}\left(\lambda_1 D\cos 2\beta\lambda_1 - \lambda_1 C\sin 2\beta\lambda_1\right) - F_1\omega\sin\theta = \dot{x}_-,$$

which gives

$$C = s - F_1\cos\theta, \qquad D = \frac{C}{\sin 2\beta\lambda_1}\left(e^{\beta c} - \cos 2\beta\lambda_1\right), \tag{5}$$

$$\sin\theta = L\dot{x}_-, \qquad \cos\theta = \frac{s}{F_1} - M\dot{x}_-,$$

$$\dot{x}_{-(1,2)} = \frac{\frac{s}{F_1}M \pm \sqrt{L^2\left(1 - \frac{s^2}{F_1^2}\right) + M^2}}{L^2 + M^2},$$

and

$$L = \frac{-0.5c(R+1)\sin 2\beta\lambda_1 + \lambda_1\left[(R-1)\cos 2\beta\lambda_1 + e^{\beta c} - Re^{-\beta c}\right]}{\lambda_1 F_1\omega\left(2\cos 2\beta\lambda_1 - e^{\beta c} - e^{-\beta c}\right)}, \tag{6}$$

$$M = \frac{(R+1)\sin 2\beta\lambda_1}{\lambda_1 F_1\left(2\cos 2\beta\lambda_1 - e^{\beta c} - e^{-\beta c}\right)}.$$

3 CONTROL NEAR THE MAIN RESONANCE

In order to find the analytical solution to equation (1) the approximate analytical method has been applied assuming that:

- the difference $x(t) - x(t-T)$ is small,
- the damping c is of the same order as the introduced formally perturbation parameter ε.

From equation (1) one obtains:

$$\ddot{x} + \alpha^2 x = \varepsilon\left\{P_0\cos\omega t + S[x(t) - x(t-T)] + Q\left[\left(1 - \frac{c}{Q}\right)\dot{x}(t) - \dot{x}(t-T)\right]\right\} \quad \text{for} \quad x < s \tag{7}$$

$$x_+ = x_-, \quad \dot{x}_+ = -R\dot{x}_- \qquad \text{for} \quad x \geq s.$$

We are looking for solution in the form:

$$x(t) = a\cos\psi.$$

We assume that the amplitude of the excitation is small and in the resonance case ($\omega \approx \alpha$) we use the method of equivalent linearization [20, 29]. The problem is reduced to the analysis of the following equivalent linear equation with the error of $O(\varepsilon^2)$:

$$\ddot{x} + 2h_e(a)\dot{x} + \alpha_e^2(a)x = \varepsilon P_0\cos\omega t, \tag{8}$$

where:

$$h_e(a) = h_e = -\frac{\varepsilon}{2}\left[\frac{S}{\alpha}\sin\alpha T - c + Q(1 - \cos\alpha T)\right],$$

$$\alpha_e(a) = \alpha_e = \alpha + \frac{\varepsilon}{2}\left[\frac{S}{\alpha}(\cos\alpha T - 1) + Q\sin\alpha T\right].$$

The solution to the equivalent linear equation (8) has the form:

$$x(t) = e^{-h_e t}(C\cos\lambda t + D\sin\lambda t) + F\cos(\omega t + \theta), \quad \text{for} \quad x < s, \tag{9}$$
$$x_+ = x_-, \quad \dot{x}_+ = -R\dot{x}_- \qquad \text{for} \quad x \ge s,$$

where:

$$F = \frac{P_0}{\sqrt{(\alpha_e^2 - \omega^2) + 4h_e^2\omega^2}}, \qquad \lambda = \sqrt{\alpha_e^2 - h_e^2},$$

and the parameters C, D, and θ are defined using the relations (5) and (6).

3.1 Stability

In order to investigate the stability we use the following approach [6]. When the vibro-impact periodic solution (9) is disturbed because of δx_l, then it causes a change of the C and D and the phase shift of δC_l, δD_l and $\delta\theta_l$, correspondingly.

The perturbated solution has the form:

$$x + \delta x_l = e^{-h_e t}\left[(C + \delta C_l)\cos\lambda t_l + (D + \delta D_l)\sin\lambda t_l\right] + F\cos(\omega t_l + \theta + \delta\theta_l). \tag{10}$$

In the above relation the time t is measured beginning from the l-th impact The next impact occurs in the time movement $t_{l+1} = 2\pi l/\omega + \delta T_l$, where δT_l denotes the period change. Because t is related to the unperturbeted equation, therefore $t_l = t + \delta t_l$, where $\delta t_l = 0$ for the l-th impact and $\delta t_l = \delta T_l$ for l+1 impact.

After transformations we get:

$$\delta x_l = e^{-h_e t}\left[(\delta C_l + \lambda D\delta t_l - h_e C\delta t_l)\cos\lambda t + (\delta D_l - \lambda C\delta t_l - h_e D\delta t_l)\sin\lambda t\right] + \\ - F(\omega\delta t_l + \delta\theta_l)\sin(\omega t + \theta). \tag{11}$$

Introducing the following boundary conditions for the l-th and (l+1)-th impact:

$$l: \qquad t = 0, \quad \delta t_l = 0, \quad \delta x_l = 0, \quad \delta\dot{x}_l = \delta\dot{x}_{l+},$$
$$l+1: \quad t = \frac{2\pi k}{\omega} + \delta t_l = 2\beta + \delta t_l, \quad \delta t_l = \delta T_l, \quad \delta x_l = 0, \quad \delta\dot{x}_l = \delta\dot{x}_{(l+1)-}, \tag{12}$$

we get the following six equations:

$$\delta C_l - F\delta\theta_l\sin\theta = 0,$$

$$-h_e\delta C_l + \lambda\delta D_l - F\omega\delta\theta_l\cos\theta = \delta\dot{x}_{l+},$$

$$\delta C_{l+1} - F\delta\theta_{l+1}\sin\theta = 0,$$

$$-h_e\delta C_{l+1} + \lambda\delta D_{l+1} - F\omega\delta\theta_{l+1}\cos\theta = \delta\dot{x}_{(l+1)+},$$

$$e^{-h_e(2\beta+\delta T_l)}\left[(\delta C_l + \lambda D\delta T_l - h_e C\delta T_l)\cos\lambda(2\beta + \delta T_l) + \right.$$

$$\left. + (\delta D_l - \lambda C\delta T_l - h_e D\delta T_l)\sin\lambda(2\beta + \delta T_l)\right] - F(\omega\delta T_l + \delta\theta_l)\sin(\omega\delta T_l + \theta) = 0,$$

$$e^{-h_e(2\beta+\delta T_l)}\Big[\big(-h_e\delta C_l-2h_e\lambda D\delta T_l+C\big(h_e^2-\lambda^2\big)\delta T_l+\lambda\delta D_l\big)\cos\lambda\big(2\beta+\delta T_l\big)+$$

$$+\big(-h_e\delta D_l+2h_e\lambda C\delta T_l+D\big(h_e^2-\lambda^2\big)\delta T_l-\lambda\delta C_l\big)\sin\lambda\big(2\beta+\delta T_l\big)\Big]+$$

$$-F\omega\big(\omega\delta T_l+\delta\theta_l\big)\cos\big(\omega\delta T_l+\theta\big)=\delta\dot{x}_{(l+1)-}.$$

Taking into account that $\delta\theta_{l+1}=\delta\theta_0+\sum_{i=0}^{l}\omega\delta T_i$, we finally get the following three equations:

$$\delta C_l-F\delta\theta_l\sin\theta=0, \tag{13}$$

$$e^{-2\beta h_e}\Big\{\delta C_l\cos 2\beta\lambda+\delta D_l\sin 2\beta\lambda+\frac{1}{\omega}\big(\delta\theta_{l+1}-\delta\theta_l\big)\Big[\big(\lambda D-h_eC\big)\cos 2\beta\lambda+$$

$$-\big(\lambda C+h_eD\big)\sin 2\beta\lambda\Big]\Big\}-\delta C_{l+1}=0,$$

$$Re^{-2\beta h_e}\Big\{\big(\lambda\delta D_l-h_e\delta C_l\big)\cos 2\beta\lambda-\big(\lambda\delta C_l+h_e\delta D_l\big)\sin 2\beta\lambda+$$

$$+\frac{1}{\omega}\big(\delta\theta_{l+1}-\delta\theta_l\big)\Big[\big(-2h_e\lambda D+C\big(h_e^2-\lambda^2\big)\big)\cos 2\beta\lambda+$$

$$+\big(2h_e\lambda C+D\big(h_e^2-\lambda^2\big)\big)\sin 2\beta\lambda\Big]\Big\}-h_e\delta C_{l+1}+\lambda\delta D_{l+1}-(R+1)\delta\theta_{l+1}F\omega\cos\theta=0.$$

The solutions are being sought in the form:

$$\delta C_l=a_1\gamma^l,\qquad \delta D_l=a_2\gamma^l,\qquad \delta\theta_l=a_3\gamma^l, \tag{14}$$

where γ denotes the constant. Substituting (14) to (13) we get the following characteristic equation:

$$b_2\gamma^2+b_1\gamma+b_0=0, \tag{15}$$

where:

$$b_2=\lambda\Big\{F\sin\theta-\frac{1}{\omega}e^{-2\beta h_e}\Big[\big(\lambda D-h_eC\big)\cos 2\beta\lambda-\big(\lambda C+h_eD\big)\sin 2\beta\lambda\Big]\Big\}, \tag{16}$$

$$b_1=e^{-2\beta h_e}\Big\{F\sin\theta\big[(R-1)\lambda\cos 2\beta\lambda-(R+1)h_e\sin 2\beta\lambda\big]-(R+1)F\omega\cos\theta\sin 2\beta\lambda+$$

$$+\frac{1}{\omega}\lambda\Big[\big(h_eC-\lambda D\big)\big(Re^{-2\beta h_e}-\cos 2\beta\lambda\big)-\big(\lambda C+h_eD\big)\sin 2\beta\lambda\Big]\Big\},$$

$$b_0=R\lambda e^{-4\beta h_e}\Big[\frac{1}{\omega}\big(\lambda D-h_eC\big)-F\sin\theta\Big].$$

The problem of the investigation of this equation stability is reduced to the analysis of the roots of the second power equation (15). If these roots fulfil the following inequality:

$$\left|\gamma_{1,2}\right|<1, \tag{17}$$

then, in accordance with the expressions (14), the solutions δC_l, δD_l and $\delta\theta_l$ approach zero at

$t \to +\infty$, and the solution will be called asymptotically stable. The above inequality is equivalent with the location of the roots inside the unit circle of the complex plane.

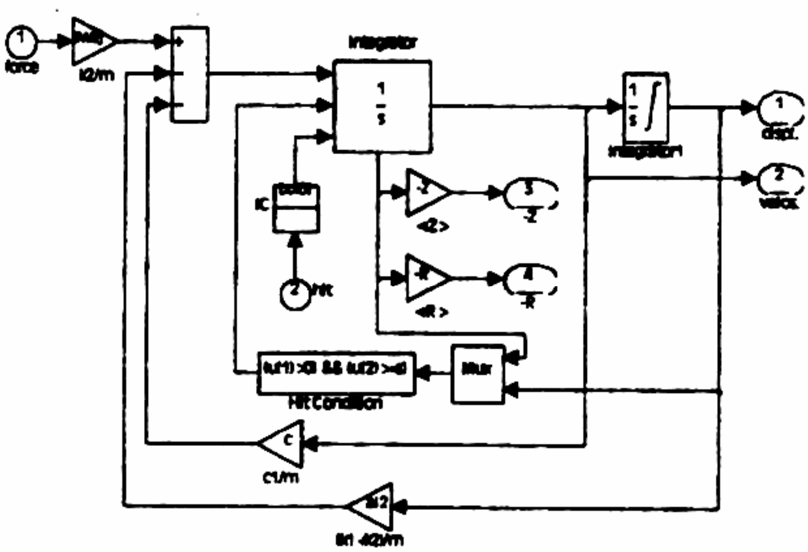
3.2 Simulation results

The figure 2 presents the simulation model of a one-degree-of-freedom system (see figure 2a) and the control of the system (see figure 2b) constructed in the MATLAB-Simlink package.

The following system parameters were used for the simulation: $m = 1[kg]$, $c_1 = 0.02[Ns/m]$, $k_1 = 7[N/m]$, $k_2 = 1[N/m]$, $\omega/\alpha = 0.99$, $y_0 = 0.01[m]$, $R = 0.65$, $s = 0.0005[m]$, and the parameters of the feedback loop: $p = 0[N/m]$ and $q = -0.01[Ns/m]$.

Figure 3 presents the simulation results in the form of phase planes and the transients of the difference $x(t) - x(t-T)$. To compare these transients, an additional μ parameter has been adopted. That parameter defines the time interval where the signal $|x(t) - x(t-T)| < \mu$. In the simulation $\mu = 10^{-6}$.

(a)



(b)

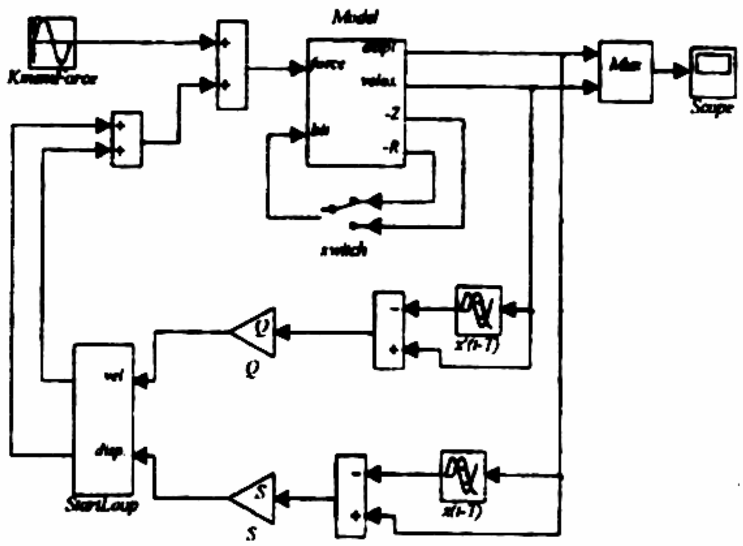


Figure 2. Simulation model of the one-degree-of-freedom vibro-impact system kinematically excited (a) and the delay loop control (b)

On the basis of the figure 3 one can conclude that the feedback loop system with a time delay stabilises more quickly (in 38 seconds) than the system without the loop (42.5 seconds). On the basis of the analysis of the stability of both systems, the following results were obtained: $|\gamma_{1,2}| = 0.635$ for the case without the loop, and $|\gamma_{1,2}| = 0.574$ for the case with the loop.

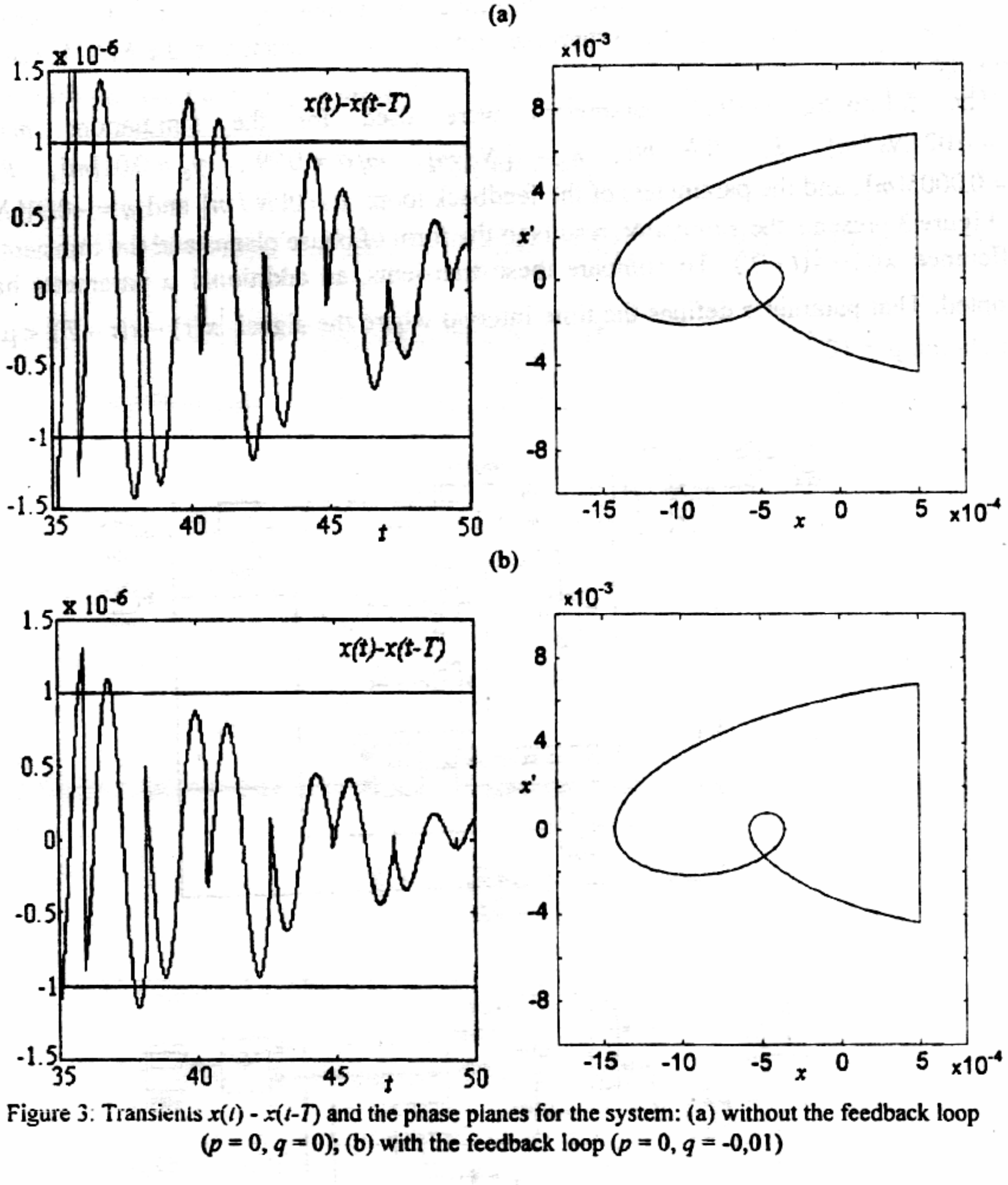


Figure 3: Transients $x(t)$ - $x(t-T)$ and the phase planes for the system: (a) without the feedback loop (p = 0, q = 0); (b) with the feedback loop (p = 0, q = -0,01)

4 TWO-DEGREE-OF-FREEDOM SYSTEM

In the figure 4 a control of a two-degree-of-freedom system with impacts with a use of the delay loop is presented.

It is obvious, that a control of the two-degree-of-freedom system can be realised in many ways. Below two fumdamental ways are mentioned:

1. A delay loop acts on the velocities and displacements related to only one of two masses;
2. Two delay loops act on the velocities and displacements of each of the masses.

In this paper we present the first possibility, where the delay loop acts on the mass m_2 (see figure 5).

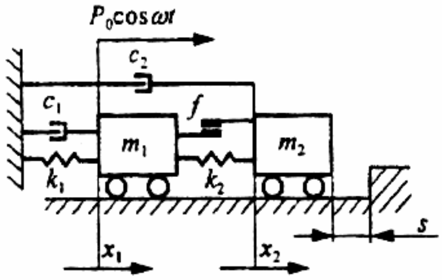


Figure 4. Schematic of the two-degree-of-freedom system (s denotes the constraint, m_i are the masses, c_i are damping, k_i are the stiffnesses coefficients and f is the friction coefficient)

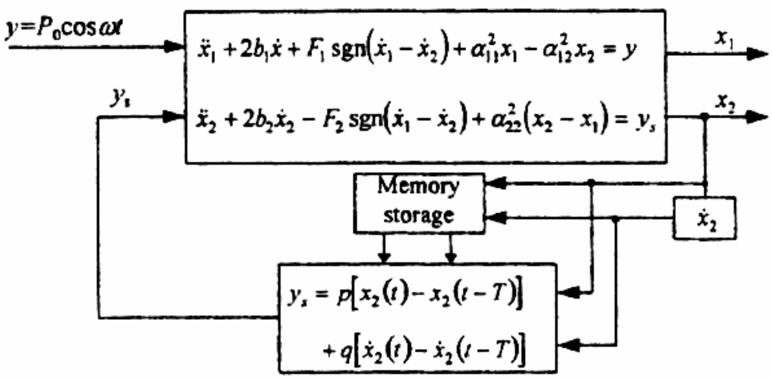


Figure 5. Schematic of control of the system presented in figure 4

The system dynamics is governed by the following equations:

$$\begin{aligned} &\ddot{x}_1 + 2b_1\dot{x} + F_1 \operatorname{sgn}(\dot{x}_1 - \dot{x}_2) + \alpha_{11}^2 x_1 - \alpha_{12}^2 x_2 = P_0 \cos \omega t, \\ &\ddot{x}_2 + 2b_2\dot{x}_2 - F_2 \operatorname{sgn}(\dot{x}_1 - \dot{x}_2) + \alpha_{22}^2 (x_2 - x_1) = S[x_2(t) - x_2(t-T)] + Q[\dot{x}_2(t) - \dot{x}_2(t-T)], \end{aligned} \tag{18}$$

where: $b_1 = c_1/2m_1$, $b_2 = c_2/2m_2$, $F_1 = f/m_1$, $F_2 = f/m_2$, $\alpha_{11}^2 = (k_1 + k_2)/m_1$, $\alpha_{12}^2 = k_2/m_1$, $\alpha_{22}^2 = k_2/m_2$, $P = P_0/m_1$, $S = p/m_2$, $Q = q/m_2$ and $T = 2\pi k/\omega$.

When the mass m_2 meets the border, we have:

$$x_{2+} = x_{2-}, \quad \dot{x}_{2+} = -R\dot{x}_{2-} \qquad \text{for} \quad x_2 \geq s. \tag{19}$$

Using (18) and (19) the simulation model of the investigated system (see figure 6), as well as its control (see figure 7) are presented using the MATLAB-Simulink package.

The following system parameters were adopted for the simulation: $m_1 = 1[kg]$, $m_2 = 0.36[kg]$, $c_1 = 0.05[Ns/m]$, $c_2 = 0.05[Ns/m]$, $f = 0.0[N]$, $k_1 = 0.64[N/m]$, $k_2 = 0.36[N/m]$, $P_0 = 0.1[m]$, $\omega = 0.8[rd\,s^{-1}]$, $R = 0.65$, $s = 0.28[m]$ and the feedback loop parameters:

a) $p = 0\ [N/m]$ and $q = 0\ [Ns/m]$ for system without delay loop,

b) $p = 0\ [N/m]$ and $q = -0.05\ [Ns/m]$ for system with delay loop.

In order to compare the simulation results the $\mu = 10^{-3}$ parameter, defined as $|x_i(t) - x_i(t-T)| < \mu$, has been introduced.

The simulation results are presented in figure 8 in a case without (a) and with (b) the delay loop, correspondingly. Without the loop the mass m_2 reaches periodic orbit after 85.5 seconds, whereas the mass m_1 after 77.5 seconds. With the loop the mass m_2 achieves the periodic orbit after 70.8 seconds, whereas the mass m_1 after the 68.5 seconds.

The calculations showed, that application of the delay loop connected with only one mass (m_2) caused an acceleration of reaching the stable vibro-impact orbit of the second mass (m_1).

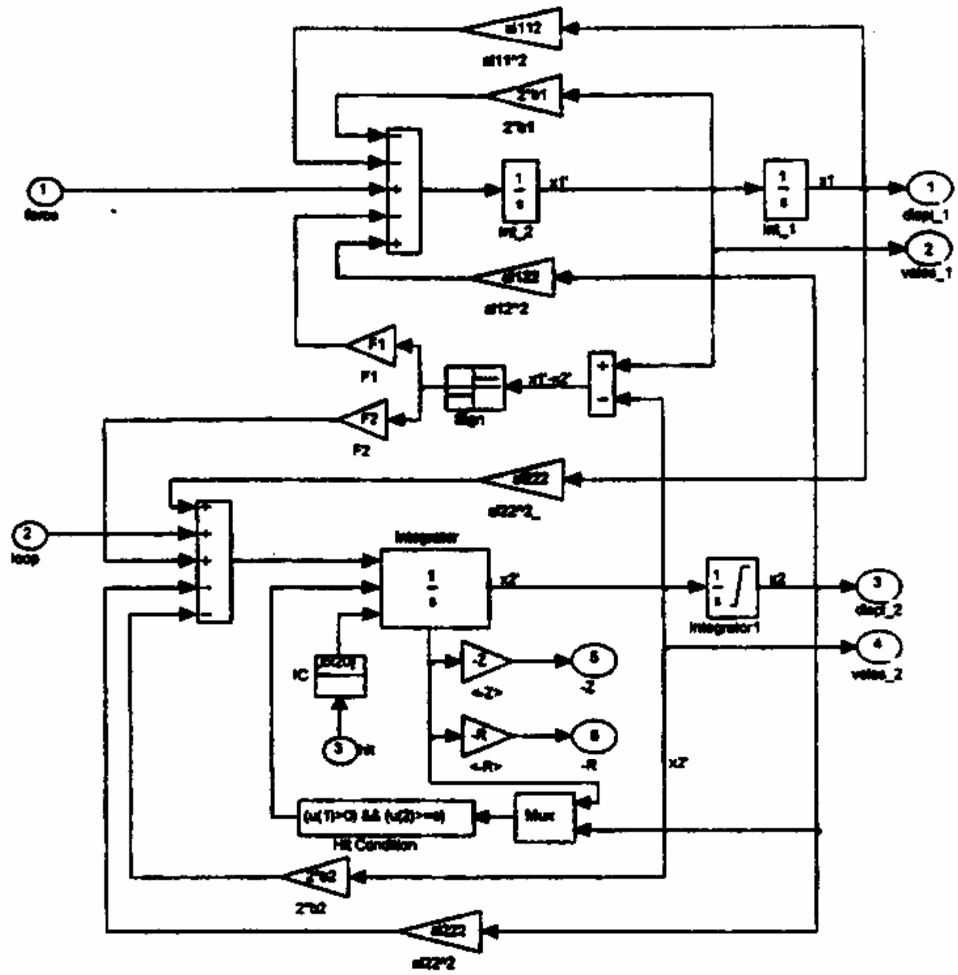


Figure 6. MATLAB-Simulink model of the analysed system

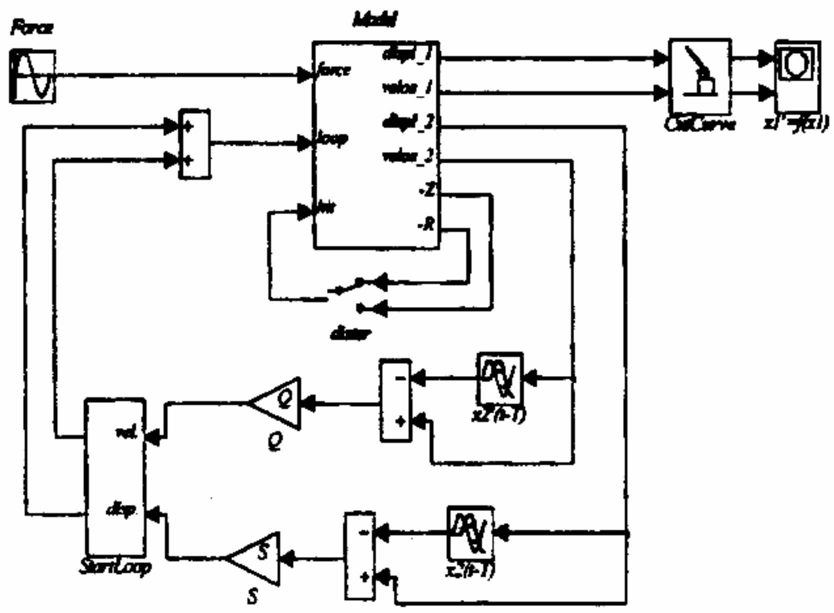


Figure 7. MATLAB-Simulink model of the control of the analysed system

5 CONCLUSIONS

In this paper we have presented an analytical approach to estimate the delay control coefficients for efficient stabilisation or destabilisation of the periodic orbit under consideration. Although the efficiency of the method is presented for $k = 1$ (periodic orbit with the same period as the excitation period) but our considerations are also valid for subharmonics (for arbitrarily taken $k > 1$). The validity of our analytical approach has been testified by numerical simulations.

To date, in the literature available to the authors, in order to achieve the mentioned objective, the feedback loop coefficients have been adopted in a random way, using the numerical observation. In this paper, in a case of one-degree-of-freedom system this problem was solved analytically for the resonance case. In addition, an efficient control of the vibro-impact two-degree-of-freedom system has been proposed and illustrated.

(a)

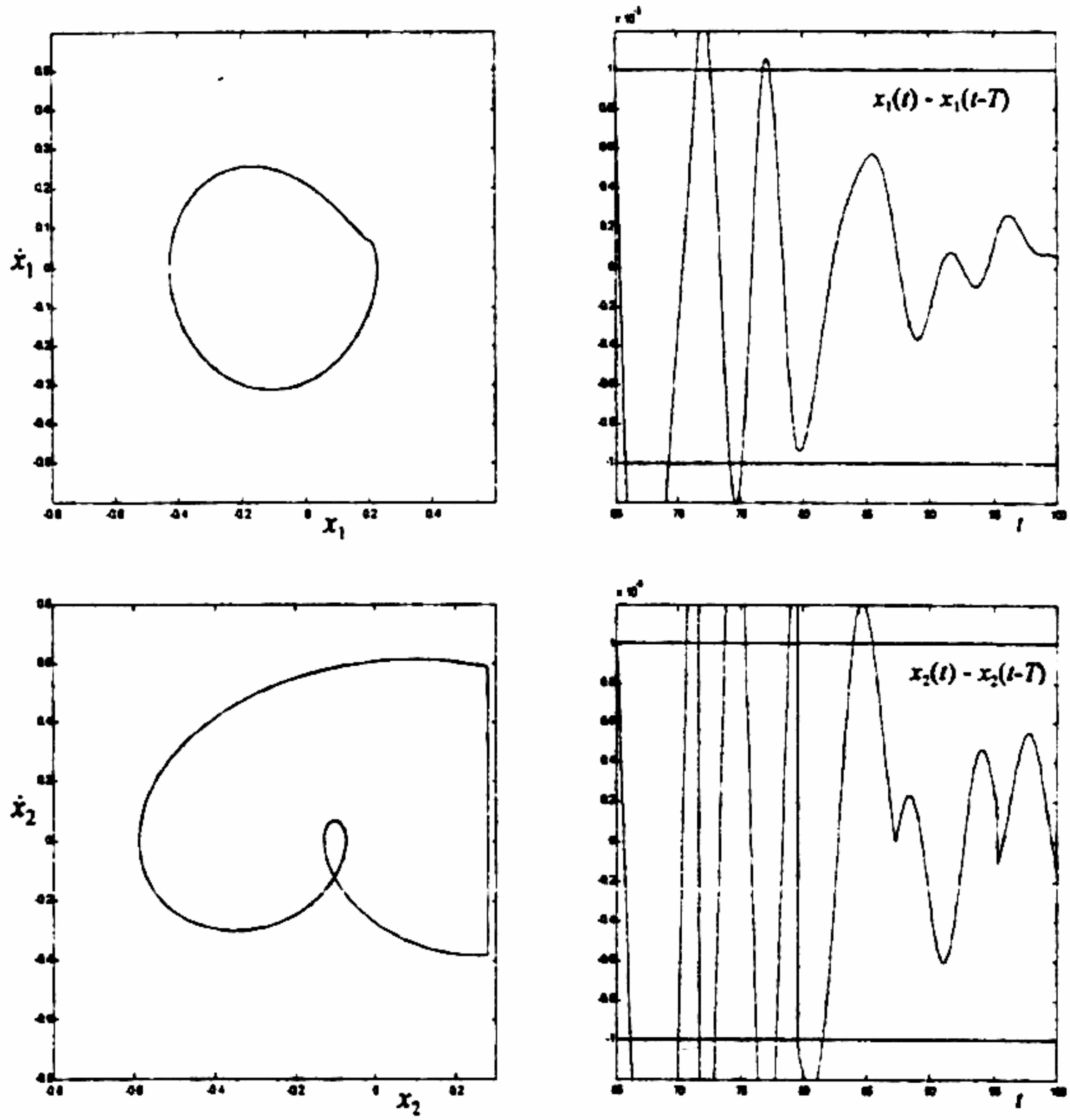


Figure 8a. Difference between two transients $x_i(t)$-$x_i(t-T)$ and phase plane approaching periodic orbit for the system: (a) without control ($p = 0$, $q = 0$); (b) with control ($p = 0$, $q = -0.05$)

(b)

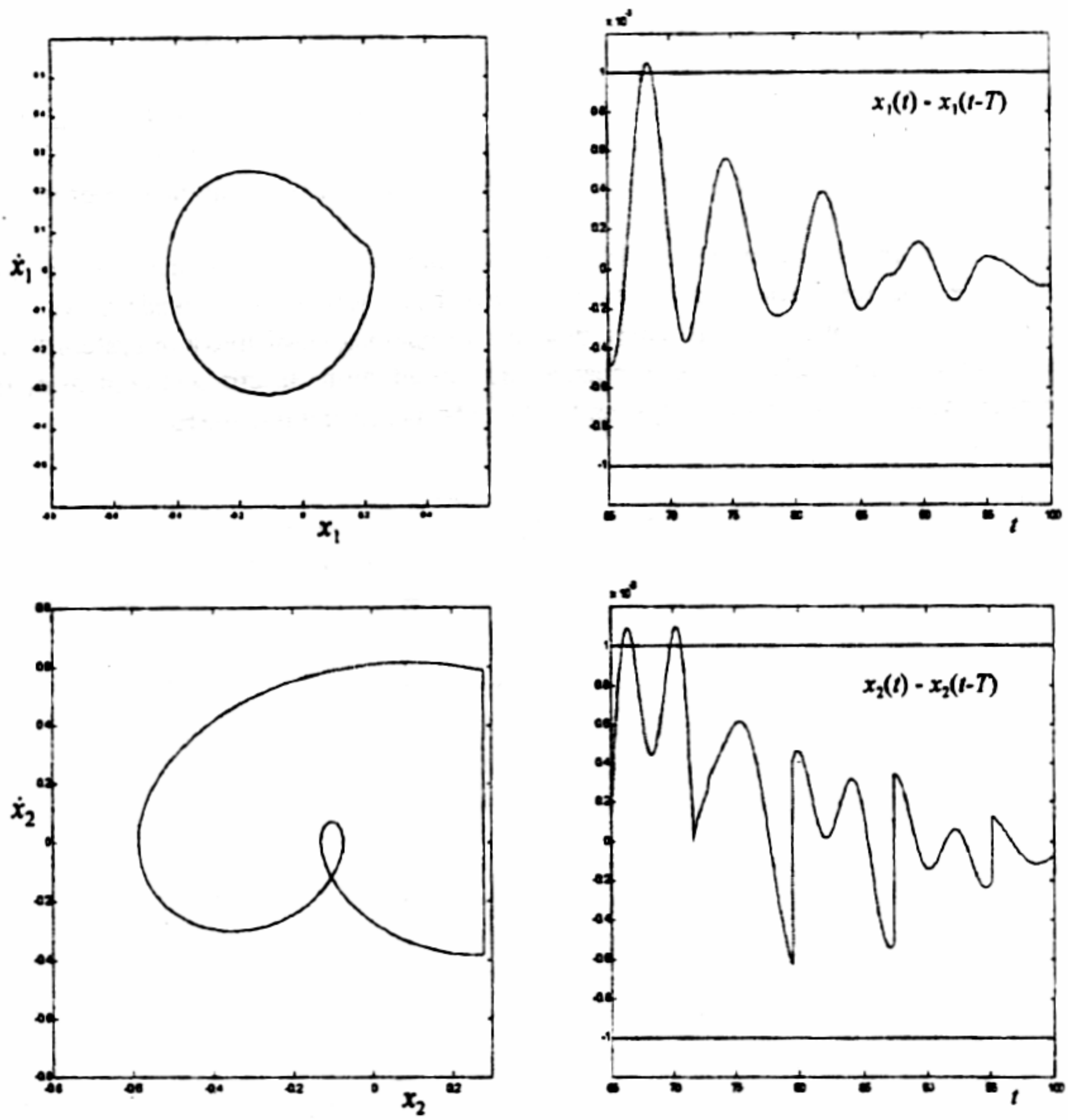


Figure 8b

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