Simulation and Design of Applied Electromagnetic Systems
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Nonlinear oscillations of a string embedded in the electromagnetic field.

Jan Awrejcewicz[a] and Tsuneo Someya[b]

[a]Technical University of Lodz, Division of Dynamics and Control (K-13), Stefanowskiego 1/15, 90-924 Lodz, Poland.

[b]Musashi Institute of Technology, Department of Mechanical Engineering, Tamazatsumi 1-28-1, Setagaya-ku, Tokyo 158, Japan

Abstract.
A general symbolic-numerical computational approach is applied to a study of the oscillations of the string type generator with a time-delay amplifier. First the analytical perturbation method (supported by symbolic computation) which yields the averaged differential equations is used and then the obtained averaged equations are analyzed numerically showing some surprising phenomena.

1. INTRODUCTION

Our report has two aims. On the one hand we analyze nonlinear dynamics of the electromechanical system, whereas on the other hand based on this example a systematic strategy of solving many other relative problems is given. The investigation includes a few steps. 1. Equations of dynamics are derived. 2. The averaging method is proposed and a written program for symbolic computation yields the averaged differential equations. 3. A further systematical study of the obtained equations is developed.

2. ANALYZED SYSTEM AND GOVERNING EQUATIONS

The electromechanical model under consideration is rather simple. It consists of a distributed mass system (string), whose oscillations are governed by a partial differential equation. The dynamics of an amplifier is modelled as a simple linear oscillator with the damping c and frequency $\sqrt{k}$. The amplifier supplies a current to the string, which is embedded into a magnetic field. The amplitude of the current undergoes control changes from the amplifier with time delay.

The electromagnetic induction B(x) acting along the string generates stresses at the ends of the string according to the following equation

$$E(t)=\int_0^l B(x)\frac{\partial u(t,x)}{\partial t}dx, \tag{1}$$

where x is a spatial coordinate, t denotes time, u(t,x) is the amplitude of oscillations of the string in the (t,x) point and l is the length of the string. The generated stresses are responsible for the force excitation

$$Y(t)=h_1E(t)-h_2E^3(t), \tag{2}$$

where h_1, h_2 are constant coefficients. Dynamics of the amplifier is governed by the equation

$$\ddot{I}(t)+2\lambda\dot{I}(t)+kI(t)=\dot{Y}(t-\mu), \tag{3}$$

where a "dot" denotes differentiation with respect to t, λ is damping coefficient, and μ is a time delay. The changes in time of I(t) and the changes in x of B(x) causes the oscillations of the string due to the equation

$$\frac{\partial^2u(t,x)}{\partial t^2}-c^2\frac{\partial^2u(t,x)}{\partial x^2}=-\frac{\epsilon}{\rho}\left(2h_0\frac{\partial u(t,x)}{\partial t}-B(x)I(t)\right), \tag{4}$$

where h_0, ρ are constants, and ϵ is a small positive parameter. The frequencies of free oscillations of the string are given by $\omega_s=\Pi cs/l$, and the homogeneous boundary conditions are as follows

$$u(t,0)=u(t,l)=0. \tag{5}$$

3. AVERAGING METHOD

For $\epsilon=0$ the solution to the equation (4) is given by

$$u_0=a_1\cos(\omega_1t+\theta_1)\sin\left(\frac{\pi x}{l}\right)+a_3\cos(3\omega_1t+\theta_3)\sin\left(\frac{3\pi x}{l}\right), \tag{6}$$

where a_1, a_3 are the amplitudes, and θ_1, θ_3 the phases.

For small enough $\epsilon\neq0$ the solution to the equation (4) is expected to be of the form

$$u=u_0+\epsilon u_1(x,a_1,a_3,\theta_1,\theta_3)+higher\ \ order\ \ terms, \tag{7}$$

We take

$$B=B_1\sin(\Pi x/l)+B_3\sin(3\Pi x/l). \tag{8}$$

For small μ the right hand side of equation (3) can be approximated by $\dot{Y}-\mu\ddot{Y}$. The solution to the linear equation (3) has a form

$$I_0(t)=M_1\cos\omega_1 t+N_1\sin\omega_1 t+M_3\cos 3\omega_1 t+N_3\sin 3\omega_1 t, \qquad (9)$$

where M_1, N_1, M_3, N_3 can be obtained and are rather complicated terms.

Further analysis is typical of the perturbation technique and details can be found elsewhere [1-3]. Because B(x) and I(t) are defined, therefore equation (4) can be solved using a classical perturbation approach. Substituting equation (7) to equation (4) and taking into account that $a_i=a_i(t)$ and $\theta_i=\theta_i(t)$ (i=1,3) the following averaged equations are obtained

$$\frac{da_i}{dt}=\epsilon P_i(a_1,a_3,\eta),$$

$$\frac{d\theta_i}{dt}=\epsilon Q_i(a_1,a_3,\eta). \qquad (10)$$

where

$$P_i=-\frac{1}{2}\frac{B_i(N_i\cos\theta_i+M_i\sin\theta_i)}{i\rho\omega_1},$$

$$Q_i=-\frac{1}{2}\frac{B_i(M_i\cos\theta_i-N_i\sin\theta_i)}{i\rho a_i\omega_1}. \qquad (11)$$

As can be seen from equation (10), we have one variable η instead of θ_1 and θ_3. This is the key point of the averaging procedure presented here. Variable η results from

$$\eta=\theta_3-3\theta_1, \qquad (12)$$

and again the averaging is applied to the equation (10) in a somewhat special way, e.g., for i=1 we take $\theta_3=\eta+3\theta_1$, whereas for i=3 we take $\theta_1=(1/3)(\theta_3-\eta)$. (It must be acknowledged that the first attempt to derive an averaged set of equations has been made by Rubanik [4]).

As it is assumed by the averaging procedure, amplitudes a_i and θ_i change with a time very slowly, and a long numerical integration to trace a behaviour of the system is required.

4. ANALYSIS OF THE AVERAGED EQUATIONS

First the stationary solutions of the equation (10) are considered. This leads to the question of solving the following nonlinear algebraic equations. To solve the problem the Powell hybrid method and an variation of Newton's method have been used. It takes a finite-difference approximation to the Jacobian with high precision arithmetics.

It is rather very difficult task to proof existance of time dependent solutions in the analyzed system of the averaged equations. The most expected situation is to find stable fixed points, which correspond to the oscillations with constant amplitudes in the original system. One of the important questions is: in which way the phase flow reaches stable fixed points, i.e. exponential or oscillatory manner. Theoretically the system can also exhibit unbounded solutions as well as stationary long time aperiodic behaviour can be found. We have found the various, and sometimes very surprising phenomena discovered during the numerical analysis.

5. CONCLUDING REMARKS

The obtained results are briefly summarized below.

1. With the increase of the B_1 constant amplitude a_1 decreases, whereas a_3 and η remain constant.
2. For each value of B_1 there are two solutions of a_1 and one of a_3 and a reach set of η solutions which possess characteristic structure.
3. In the case of time dependent solutions the occurrence of time delay causes loss of stability of $a_1 = a_3 = 0$ and sudden jump to the constant positive values, where they remain constant in spite of $\theta_4(t)$ time evolution (the new frequency has $\dot{\theta}_3$ occurred).
4. Amplitude a_1 is constant, whereas amplitude $a_3(t)$ monotonously decreases in time, and the phases remain constant (time independent).
5. A transitional phase modulated state can be realized.
6. After a passage with ω through resonance a sudden extinguishing of oscillation occurred.
7. Unbounded solutions exhibited by a_3 has been found.
8. Steady state with the oscillation of a_1, a_3, θ_3 and constant $\dot{\theta}_1$ is also detected.

6. REFERENCES

1 I. G. Malkin, Some Problems in the Theory of Nonlinear Oscillation, Nauka, Moscow, 1956, in Russian.

2 V. I. Zubov, Theory of Oscillations, High School, Moscow, 1979, in Russian.

3 J. Guckenheimer and P. Holmes, Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields, Springer, New York, 1983.

4 W. P. Rubanik, Oscillations in Complex Quasilinear Systems with Delay, University Press, Minsk, 1985, in Russian.