Nonlinear Phenomena in Electromagnetic Fields
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Identification of stiffness and damping coefficients of the rotor supported in electromagnetic bearings

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Abstract

This paper presents some ideas concerning the control of the complex rotating machinery systems as well as the identification of their damping and stiffness coefficients based on a modal approach. First, a method to control both linear and nonlinear rotating systems is outlined, and then a two-degrees-of-freedom system is used as an example to illustrate a proposed identification procedure using a phase resonance method.

1. INTRODUCTION

There are various examples of application of electromagnetic phenomena in order to excite or control vibration in rotating mechanical systems. For example, magnetic bearings are used to control flexible rotors [1,2], a sliding mode control is adopted for the effective control of structure vibration and magnetic bearings [3,4], or active dynamic absorbers supported by the electromagnetic linaear actuators are proposed [5].

The electromagnetic forces are often nonlinear and coupled with the air gap. In order to obtain effective results during the control of machines influenced by electromagnetic fields, the identification of the all system is important. It allows us to set up a mathematical model of the system analyzed.

We consider further the magnetic bearing systems which are often controlled by PD or PID type controllers tuned based on errors and a knowledge of the governing equations, which considerably saves the time of finding the appropriate tuning of

PD or PID controller.

A proposed extended method allows for modal control of rotating masses joined on a flexible rotor supported by electromagnetic bearings, that is an application of the PID (analog or digital) controller leads to a considerable reduction of the vibrations of the whole system analyzed.

The use of a modal approach is particularly effective. The first method introduced in the next section, for control of the linear and nonlinear rotating systems can be successfully applied also to identification of properties of these systems. Additionally, there are some other benefits of a modal approach, which are briefly summarized as: (a) eigenfrequencies and modal configurations of the system are found; (b) governing equations are derived; (c) problems with so called "bad conditioned" matrices are avoided.

2. THEORETICAL APPROACHES

We begin with an extension of the modal control algorithm presented in [1] to the more general rotating systems. Consider a system governed by the following ODE's

$$M\ddot{x} + C\dot{x} + Kx = F, \tag{1}$$

where: x is the displacement vector; M, C, K are the mass matrix, damping matrix and stiffness matrix respectively, and f is the electromagnetic bearing-force vector. Applying a modal matrix $\Phi(x = \Phi q)$, the rows of which are equal to the eigenvectors of the matrix $M^{-1}K$, we obtain

$$\mathbf{m}\ddot{\mathbf{q}} + \mathbf{c}\dot{\mathbf{q}} + \mathbf{k}\mathbf{q} = \mathbf{f}, \quad \mathbf{f} = \Phi^T F. \tag{2}$$

Now the matrices $\mathbf{m}$ and $\mathbf{k}$ are diagonal, but $\mathbf{c}$ is in general not diagonal. In practice, however, the values of $\mathbf{c}$ are small in comparison to $\mathbf{m}$ and $\mathbf{k}$ and therefore an approximate diagonal matrix can be used, as below. As a result we obtain the quasinormal ODE's

$$m_i \ddot{q}_i + c_i \dot{q}_i + k_i qsubi = f_i, \quad i = 1, \ldots, n. \tag{3}$$

Each of the above equation governs the dynamics of the i-mode. Using the digital controller D_i to each of the modes independently, the modal controlled signals are then transformed into the physical magnetic forces

$$F_i = \Phi_i^{-T} D_i q_i. \tag{4}$$

Thus, a suitable controller PD or PID can be used in order to reduce of the i-modal resonancy independently. The approach presented can also be applied to nonlinear rotating systems, governed by the following equations

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \epsilon\mathbf{C}(\mathbf{x},\dot{\mathbf{x}}) + \epsilon\mathbf{F}\sin\omega t. \tag{5}$$

The modal matrix Φ is defined as

$$\mathbf{M}^{-1}\mathbf{K}\Phi_i = \omega_i^2\Phi_i, \tag{6}$$

and ω_i are eigenfrequencies obtained from (5) for $\epsilon = 0$. By the analogy to the first consideration we find

$$\mathbf{m}\ddot{\mathbf{q}} + \mathbf{k}\mathbf{q} = \epsilon\mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) + \epsilon\mathbf{f}\sin\omega t. \tag{7}$$

where:

$$\mathbf{m} = \Phi^T\mathbf{M}\Phi, \quad \mathbf{k} = \Phi^T\mathbf{K}\Phi, \quad \mathbf{f} = \Phi^T\mathbf{F},$$

$$\mathbf{c}(\mathbf{q},\dot{\mathbf{q}}) = \Phi^T\mathbf{C}(\phi\mathbf{q},\Phi\dot{\mathbf{q}}). \tag{8}$$

In this case the normal modes are coupled because of the terms, which describe the influence of all nonlinearities and dampings of the system reduced to each independent mode of oscillation. The vector **f** defines the effect of influence of the sinusoidal forces for each normal mode. To reduce each of the modal resonances we take $\omega = \omega_i$. This implies that the norm of a solution of i-th equation of (7) will be one order greater in comparison with the solutions of the other equations. It is therefore necessary to consider only the following equation

$$m_i\ddot{q}_i + k_i q_i = \epsilon c_i(q_i,\dot{q}_i) + \epsilon f_i \sin\omega_i t, \tag{9}$$

where now c_i includes the nonlinear effect of all nonlinearities for the i-mode. Introducing digital controllers in a similar way to (4) we can damp the resonant mode oscillations.

Assuming that the vibrations are enough small and the linearized rotor equations are governed by two second order ODE's we present a method for their identification.

Consider the following equations of motion of the rotor

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix}, \tag{10}$$

where $P_1 = P_2 = 0$ and m_1, m_2 are known. In order to realize the identification we use the harmonic excitation

$$P_k = (P_{kR} + iP_{kI})e^{i\theta_k}e^{i\omega t}, \quad (k=1,2) \tag{11}$$

and the solutions of (10) are sought in the form

$$x_k = (U_k + iV_k)e^{i\omega t}, \quad (k=1,2). \tag{12}$$

A phase resonance condition leads to the very efficient equations

$$\begin{pmatrix} k_{11} - \omega^2 m_1 & k_{12} \\ k_{21} & k_{22} - \omega^2 m_2 \end{pmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{13}$$

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} P_{1R} \\ P_{2R} \end{Bmatrix}. \tag{14}$$

The eq.(13) has the same form as the conservative systems (when $c_{11} = c_{12} = c_{21} = c_{22} = 0$ in (10)).Thus, a phase resonance appears if the exciting frequency is equal to the eigenfrequencies. Eq. (14) shows how all resistance forces are compensated by the external forces.

3. CONCLUSIONS

Some efficient methods to control and identify rotating machinery elements supported in electromagnetic bearings are proposed. All are based on the modal oscillation approach which leads to a simple and convenient choice of the controller type, or the choice of measurement technique in the case of identification. In the last case a phase resonance condition is realized which leads to two uncoupled sets of equations. For $\omega = \omega_1$ and $\omega = \omega_2$ measuring only the amplitudes u_k, P_{kR} (k=1,2), the unknown stiffness and damping coefficients can be calculated.

4. REFERENCES

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