



MONOGRAPHS

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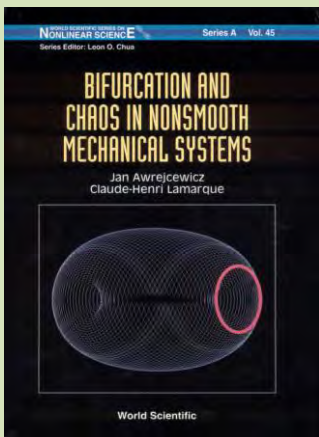
Bifurcation and Chaos in Nonsmooth Mechanical Systems

(with C.-H. Lamarque)

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SUMMARY



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The material of the book can be treated as interaction of the following key word fields: Oscillations, impacts, friction, modern theory of dynamical systems, applied mathematics, numerical techniques, piecewise linear and non-linear systems. In the standard terminology we deal with the lumped, mechanical systems governed by ordinary differential equations.

One may say that our attention is focused on old and classical problems on mechanics, i.e., impact and friction. Yes, this is true and this is a motivation to write the book. We principally are followers of original sources of mathematics and mechanics and vice versa. The applied mathematics took their many ideas from the natural sciences like mechanics, and mechanics looked carefully for development of mathematics to solve many of (still) unsolved problems. Both key problems roughly expressed by the terms "impact" and "friction" are still unsolved and require idea from mathematics for their solution. These ideas can be easily followed in our book. We try to attack old natural problems using more sophisticated approaches from the points of view of applied mathematics and computational physics (mechanics).

We use all recent developments in the field of modern dynamical systems (bifurcation and chaos theories, theory of differential inclusions, both modern analytical approaches to predict chaos and numerical algorithms to solve the "discontinuous" problems which are sometimes rigorously discussed, etc.) to look again for "impact" and "friction". The pendulum is taken as another paradigm tool that exhibits fascinating and complicated non-linear behavior. The exhibition of simultaneous simplicity and complexity in nonlinear dynamical behaviour is one of the main goals of the reported research.

The presented book is focused on mechanical (natural) insight than on overloading a reader with "heavy" mathematical treatment. However, we do not forget on mathematics addressing some questions in a more rigorous manner. We provide the reader with many illustrations and physical explanations of the system behaviour, and we try to use the most simplest objects to exhibit the most complicated discontinuous dynamics.

The book is intended to be used by physicists, applied mathematicians and engineers and it can also serve as a textbook for a class on nonlinear oscillations. Our simple and clear approaches yield many fascinating discoveries in the ocean of non-linear (piecewise linear) lumped dynamical system behaviour. Our examples taken from mechanics serve as a call to researchers: be patient, do not always believe in authorities, utilize mathematics since the fascinating non-linear hidden world waits for your discovery!

In Chapter 1 both Fillipov's and Aizerman's theories are described which are suitable

to analyse piecewise continuous systems. Two simple examples introduce a reader (very often met in physical systems) to jumps of velocity and acceleration. Moreover, a boundary value problem is formulated to find analytically and numerically oscillations of one-degree-of-freedom non-autonomous oscillator with Coulomb friction.

Chapter 2 deals with the mathematical background for multivalued formulations. First the origin of both smooth and unsmooth nonlinearities is illustrated. Then two simple model examples with friction and impact are studied. The existence and uniqueness problems, maximal monotone operators and ill posed problems are rigorously discussed. Finally, some remarks concerning both deterministic and stochastic differential equations are given.

In Chapter 3 well-adapted numerical schemes to trace differential inclusions and differential equations are proposed. Since the book is intended to serve as a text for engineers and applied scientists, the methods and language heavily accessible to engineers are avoided, and hence a rigorous mathematical treatment is here omitted.

Chapter 4 is devoted to general questions of convergence of numerical schemes for relatively large classes of nonlinear problems governed either by differential inclusions or by ordinary differential equations and differential inclusions. Dynamical systems with friction (or elastoplastic terms) and impacts are studied. A special attention is paid to resolution methods, sticking motion and accumulation of impacts. More rigorous mathematical background including theorems and their proofs applied to numerical analysis of nonsmooth differential systems is included.

Bifurcation and chaos of a particular van der Pol-Duffing oscillator with Coulomb friction is discussed in Chapter 5. The so-called van der Pol averaging procedure is applied to trace "0"-type and complex bifurcations and their correspondence to chaos is discussed.

A systematic approach to trace stick-slip dynamics with friction using mechanical example with two-degrees-of-freedom is presented in Chapter 6. A special attention is paid to analyse transitions from the stick to slip, slip to stick and slip to slip states.

The shooting method is used to trace stability and bifurcations of equilibria and periodic orbits. Many interesting examples of bifurcations and chaotic behaviour of investigated slide-roll oscillator are reported and discussed.

Another two-degrees-of-freedom system with horizontally situated bodies and Coulomb friction is studied in section 6.3. Some standard numerical approaches to analyse nonlinear and discontinuous dynamics are reviewed and a validity of analytical approximation to friction is discussed. A variety of non-linear dynamical phenomena is reported.

The piecewise linear approximations are discussed in Chapter 7. First exact and approximated models are analysed, and then numerical results are given. This chapter is based on reference.

Chapter 8 is devoted to analysis of Chua circuit, and particularly to electromechanical or purely mechanical realization of Chua's circuit and Chua's unfolding circuit. In spite of some proposed mechanical devices governed by Chua's equations the rheological model of the generalized double scroll Chua's circuit is proposed, and existence and uniqueness of solutions of discontinuous Chua's model are addressed.

The modal approaches to investigate one- and two-degrees-of-freedom mechanical systems with impacts are proposed in Chapter 9. A piecewise exact integration is used to study the periodic responses under sinusoidal excitation. Both weak and strong coupling are examined while studying two-degrees-of-freedom systems. The source of this chapter is and.

An approximate solution of the differential inclusion model of a pendulum with friction is analysed in Chapter 10. Existence and uniqueness of results and numerical schemes convergence is studied. Special coexisting attractors generated by friction are investigated. The Melnikov's method is used to predict chaos.

In Chapter 11 several types of rheological models constituted of springs, St-Venant elements and dashpots are examined within a frame of a differential inclusion. Numerous simulations exhibiting hysteresis limit cycles are presented for different periodic forcings.

Seven-degrees-of-freedom mechanical system with impacts is studied in Chapter 12. It models a cable with its shaft and a gear level with its support. The numerical scheme, numerical results and comments are given.

In Chapter 13 dynamics of grazing periodic solutions with one impact at zero velocity per cycle are investigated. Stability of grazing periodic solutions are exhibited and illustrated, among others.

The triple pendulum with damping external forces and with impacts is investigated in Chapter 14. The theory of Aizerman and Gantmacher is applied to calculate the fundamental solution matrices in the system with discontinuities. The fundamental matrices are used during calculation of the Lyapunov exponents, during stability analysis of periodic solutions (Floquet multipliers) and in shooting procedure. Periodic, quasi-periodic and chaotic dynamics are reported and discussed.

For the first time in 1999 stick-slip chaotic dynamics has been predicted in a one-degree-of-freedom quasi-autonomous oscillator using the Melnikov's method. Chapter 15 is based mainly on the mentioned reference. The critical chaotic threshold curves, where infinitely small external periodic perturbations may lead to chaos, are reported, among others.

In the Chapter 16 the plane axially-symmetric problem dealing with thermoelastic contact of the rotating shaft with the rigid bush fixed elastically to the steady base by springs in conditions of frictional self-oscillations and wear is studied. This problem is rather complicated, since a shaft temperature, the contact pressure, wear of the bush, and the bush movements are coupled with each other creating a frictional joint. Steady state solutions with and without wear are analysed and some important conclusions are derived.

In the last Chapter 17, a control procedure for systems including elasto-plastic terms is described: It is based on classical Riccati process obtained for linear systems. A linear system is associated with non linear initial one. Then control force is applied to the full non linear system.

This book project is a result of our seventh years co-operation, which started in 1996 with the help of the Région Rhone-Alpes (France). In September 16-18, 1996 the International Conference on "Nonlinearity, Bifurcation and Chaos. The Doors to the Future" has been organised in the frame of co-operation between the Technical University of Łódź (Poland) and Ecole National des Travaux Publics de l'Etat of Vaulx-en-Velin (France). This conference has attracted many top researches from the field.

Then our common research has been supported by numerous grants from both Polish and French sides ("Tempra" Program in 1995-1996, and "Tempra" PECO Program in 2000-2001), including also common grants (like three years grant "Polonium" in 1999-2001).

Also many of our Ph. D. students and co-workers have been involved in the cooperation in the frame of Erasmus/Socrates Programs in the years 1996-2001.

Our cooperation has been continued and re-freshed by the every two years meetings during the series of international conferences "Dynamical Systems - Theory and Applications" sponsored by the Technical University of Łódź, Ministry of Education of Poland (KBN) and the Stefan Batory Foundation (Poland). Also the grant obtained from the Ministry of Education of France in 1999 is highly appreciated (J. A.)

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This book is also the result of common works with several of former students (Vincent Mesnier, Tristan Robert) or former Ph. D. students of Claude-Henri Lamarque: Jérôme Bastien, Olivier Janin, Stéphane Pernot. Without them many things should have not been possible. And even if they are not directly involved in this book, one of us (CHL) sends particular thought to his friends and colleagues L. Jézéquel, V. Roberti, J.-M. Malasoma, J.-M. Cornet, F. Roberti. Especially thanks are addressed to Michelle Schatzman (Laboratory MAPLY, Lyon I University and UMR CNRS 5585) for common works and discussions. Her help in the mathematical (but not only) point of view is always invaluable.

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